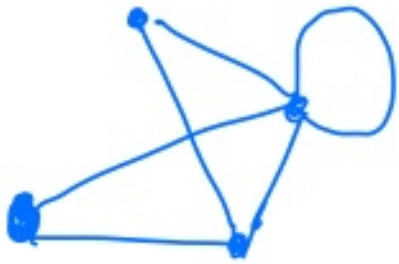
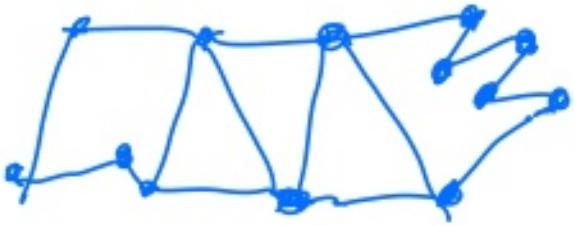
Graphs

Examples



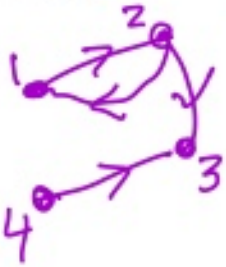


Ingredients of a graph $G = (V, E)$,

V = set of all vertices (points)
(labels) ~ could be infinite

E = set of edges

For a directed graph, where every edge has a direction,



E = set of ordered pairs

(u, v)

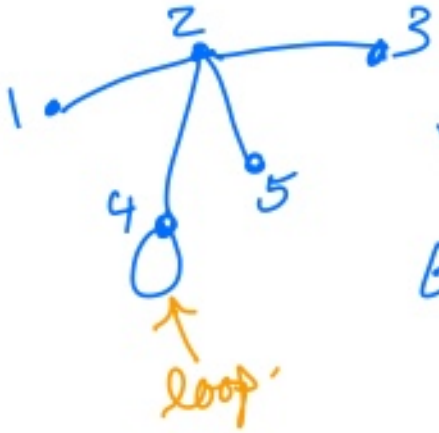
initial vertex (start) — terminal vertex (end)

$V = \{1, 2, 3, 4\}$

$E = \{(4,3), (2,1), (1,2), (2,3)\}$

For an undirected graph, where the edges are not directed, E = set of unordered pairs

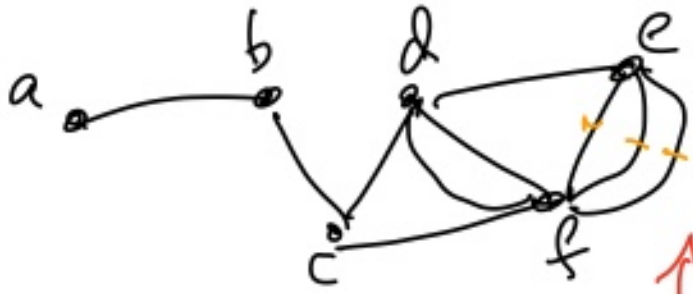
$\{u, v\} = \{v, u\}$



$V = \{1, 2, 3, 4, 5\}$

$E = \{\{1,2\}, \{2,4\}, \{2,3\}, \{3,5\}, \{4,4\}\}$

a loop



← 3 edges that are $\{e, f\}$.

when this happens, we call it a multigraph.

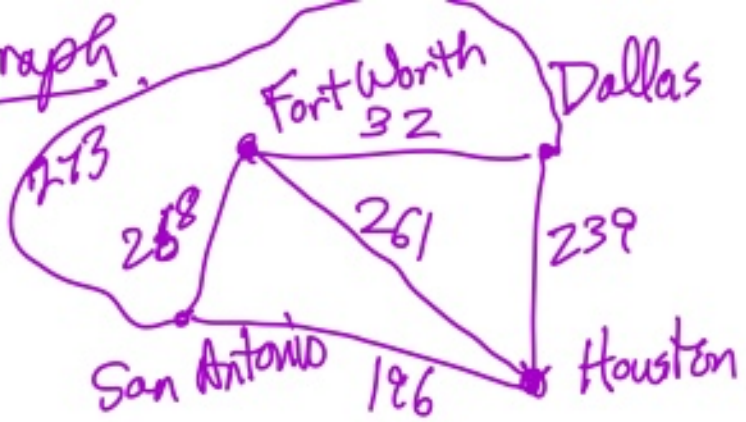
We call a graph a simple graph if no edge appears twice, and there are no loops.

Sometimes, if a graph has multiple edges & loops, it's called a pseudograph.

Weighted Graph (could be weighted & directed or weighted & undirected)

means there is a number ("the weight") associated to each edge, i.e. there is a weight function $w: E \to \mathbb{R}$.

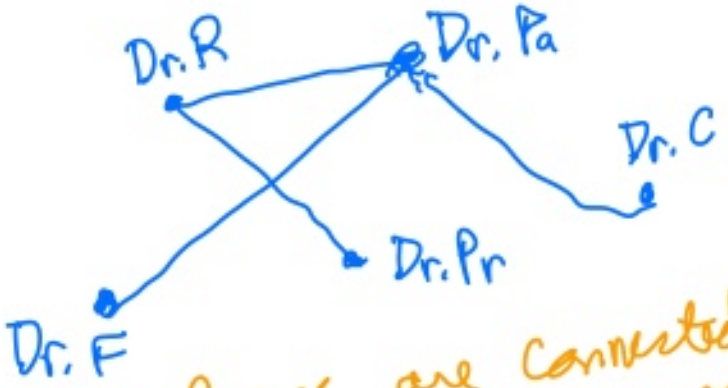
Example of weighted graph.



here the weights are the highway distances between the cities.

Examples of Graph Models.

- Collaboration Graph.



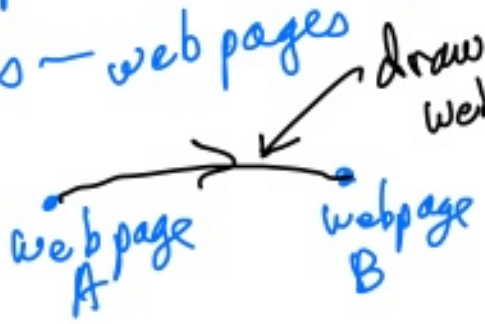
Two professors are connected if they've written a paper together.

- Computer Network

— nodes — servers/data centers

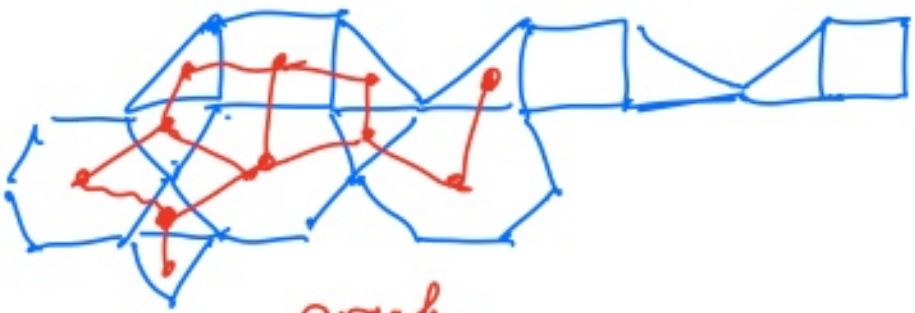
- Web graph — directed graph

vertices — web pages



draw edge if webpage A links to webpage B.

- Tiling graph — tessellation of the plane with polygons



graph.

vertices — polygons

edges — two polygons are connected if they share an edge border.

- Road network —